Generalized Polygonal Basis Functions for the Electromagnetic Simulation of Complex Geometrical Planar Structures

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***Abstract* — This paper describes the generalization of the well known rectangular and triangular rooftop functions to polygonal subdomains. The rooftop functions are commonly used for the discretization of planar currents in electromagnetic simulators. The new generalized polygonal functions allow for a more efficient meshing of complex geometrical structures in terms of polygonal shaped cells. They naturally model the current flow in the polygonal cells, satisfy the current continuity relation and their usage significantly enhances the electromagnetic simulation performance for complex geometrical structures. The increased simulation perfomance is demonstrated for a complex RF board interconnection layout.**

I. INTRODUCTION

Over the past decade, planar electromagnetic simulators [1]-[4] have been extensively used for the time-harmonic characterisation of planar structures in RF board and microwave circuit and antenna applications. The electromagnetic behaviour of the planar structure is governed by an integral equation in the unknown surface currents flowing on the planar metallization patterns. This integral equation is solved numerically by applying the method of moments. The planar structure is hereby subdivided or discretized into a mesh of rectangular and/or triangular cells. Subsectional vector basis functions defined over these cells are used for the modelling of the surface current distribution. Due to the current continuity condition, the normal component of the discretized current needs to be continous across the boundaries of adjacent cells in the mesh. This follows from the observation that unphysical Dirac line charges have to be avoided in order to have a physical acceptable numerical solution. The tangential component of the current however is allowed to jump across the intercell boundaries.

A set of basis functions for which these requirements are fulfilled are the well known rooftop functions defined over a rectangular or triangular subdomain [5]-[7]. With each side of the cell, one such function is associated that models the normal component of the current flowing across the cell side. This function is constant along the corresponding side and varies linearly to zero along the adjacent sides of the cell. Rooftop functions with rectangular subdomain have only one component in the direction normal to the correponding cell side. Rooftop functions with triangular support however also have a component tangential with the cell side. This component is necessary to obtain the continuity of the normal current across the adjacent cell sides in the triangular cell.

In this paper, we describe the generalization of the rectangular and triangular rooftop functions to polygonal shaped subdomains. These generalized polygonal vector functions naturally model the current flow in a polygonal cell and by definition satisfy the normal current continuity condition across the edges of the cell. When used as current basis functions in a method of moments numerical solution algorithm, they significantly enhance the simulation performance for complex geometrical structures.

II. GENERALIZED BASIS FUNCTIONS

The commonly used triangular and rectangular rooftop functions are locally curl-free with locally constant charge density. In order to generalize these functions to cells with a more general shape, the pertinent question is: can we find a curl-free current density **J** with constant divergence $\nabla \cdot \mathbf{J} = A$ over a general domain D such that its flux $\mathbf{J} \cdot \mathbf{n} = \phi(s)$ has pre-assigned values on the boundary c. In other words

$$\frac{\partial}{\partial x} J_x + \frac{\partial}{\partial y} J_y = A \qquad (1)$$

$$\frac{\partial}{\partial y} J_x - \frac{\partial}{\partial x} J_y = 0 \qquad (2)$$

in D and

$$\mathbf{J} \cdot \mathbf{n} = \phi(s) \qquad (3)$$

on the boundary c. The curl and div conditions imply that

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$$\mathbf{J}(\mathbf{r}) = \frac{A}{2}\mathbf{r} + \nabla K \qquad (4)$$

where K is a harmonic function, i.e. a solution of the Laplace equation $\nabla^2 K = 0$ in D. The Neumann boundary condition is

$$\frac{\partial}{\partial n} K = \phi(s) - \frac{A}{2}\mathbf{r}\cdot\mathbf{n} \equiv \xi(s) \qquad (5)$$

on c. Note that $AS = \oint \phi(s)\, ds$, where S is the area of D. From potential theory [8] we know that K admits the solution

$$K(\mathbf{r}) = \oint \ln |\mathbf{r} - \mathbf{r}(s)| \sigma(s)\, ds \qquad (6)$$

where σ(s) represents an unknown boundary source distribution. The gradient of K inside D is

$$\nabla K = \oint \{\nabla \ln |\mathbf{r} - \mathbf{r}(s)|\} \sigma(s)\, ds \qquad (7)$$

On the interior boundary $c\_$ the gradient of K is [9]

$$\nabla K = PV \oint \{\nabla \ln |\mathbf{r}(s) - \mathbf{r}(s')|\} \sigma(s')\, ds' - \pi \mathbf{n}(s)\, \sigma(s) \qquad (8)$$

This leads to the principal value integral equation for σ(s)

$$PV \oint \left\{ \frac{\partial}{\partial n} \ln |\mathbf{r}(s) - \mathbf{r}(s')| \right\} \sigma(s')\, ds' - \pi\sigma(s) = \xi(s) \qquad (9)$$

The vector field $\mathbf{J}(\mathbf{r})$ is completely defined by $\phi(s)$ by means of equations (4), (5), (7) and (9). Applied to a general polygonal domain, when we take $\phi(s)$ = 1 on one side of the polygon and $\phi(s)$ = 0 on the other sides, we obtain a generalization of the well-known triangular and rectangular vector functions. Of course, these vector functions must be combined into doublets (rooftops) in order to guarantee the normal continuity of the current density. Figures 1 and 2 show vector plots of some generalized basis functions calculated for a general polygonal cell as well as for a more specific T-shaped cell. It is clear that these functions naturally model the current flow in the polygonal cells.

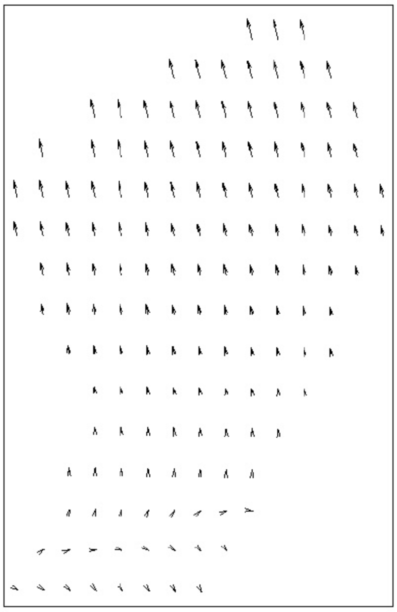

Figure 1. Generalized basis function for polygonal cell.

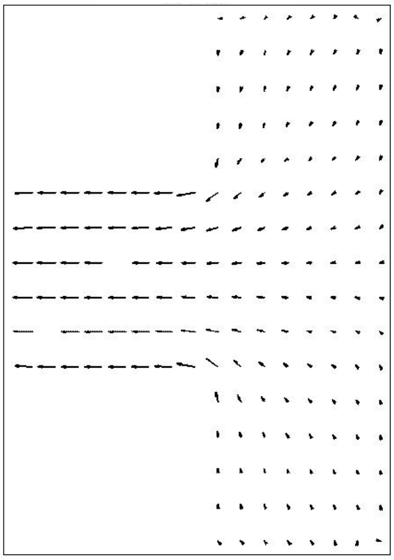

Figure 2. Generalized basis function for T-shaped cell.

III. ELECTROMAGNETIC SIMULATION

We have applied the generalized polygonal vector functions in the method of moments simulation to model the surface current distribution of complex geometrical structures. The planar electromagnetic technology behind this simulation and the mesh reduction algorithm used to construct the polygonal mesh is described in another paper [10]. We only whish to point out here the main benefits of using the generalized polygonal basis functions in this process.

It is known that to obtain accurate results, the size and the number of cells needed in the mesh is mainly determined by the electrical wavelength. Typically with rooftop basis functions, at least 10 subdivisions per wavelength are needed. For complex geometrical structures, the performance of the solution process is limited by the mesh generation process. Being able to only use rectangles and triangles leads to meshes in which the cell density is mainly determined by the geometrical detail and not by the electrical wavelength. As a result, the mathematical system is overdimensioned and the required computer time and memory resources in the solution process can be very high.

The main benefit of using a polygonal meshing lays in the elimination of the redundancy created by the rectangular/triangular meshing. The use of polygonal cells enhances the flexibility of the meshing and reduces the number of cells needed to discretize a given complex geometrical pattern with a given mesh density. It allows to generate minimal meshes that comply to the wavelength criterium imposed on the mesh density and with a number of cells independent of the geometrical complexity. This property leads to significant performance improvements in the electromagnetic simulation when compared to the traditional rectangular/triangular meshing.

IV. NUMERICAL EXAMPLE

The example we used to demonstrate the enhanced simulation performance is shown in Figure 4. It consists of the interconnection layout for an RF board circuit (figure 4). The lumped components are removed from the board and replaced by port connections, resulting in a total number of 60 ports. The interconnection structure is meshed at 1 GHz with an imposed mesh density of 20 cells per wavelength. In order to compare, we first used a rectangular/triangular meshing algorithm. The resulting mesh is shown in figure 5 (a). The corresponding interaction matrix has a size of 3428. Due to the geometrical complexity this mesh contains a lot of redundant elements. The polygonal mesh corresponding to the imposed mesh density (figure 5(b)) gives a much smaller interaction matrix size of 733. The simulation statistics for the two simulations are compared in table 1. The polygonal mesh yields a 3-fold memory reduction and a 14-fold speed improvement in the electromagnetic simulation. Some of the simulated S-parameters up to 1 GHz for the 60-port interconnection layout are displayed in figure 6 for both the rectangular/triangular mesh and the polygonal mesh. The results are identical.

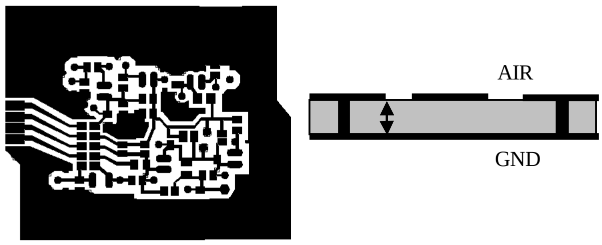

Figure 4. Interconnection layout and dielectric substrate.

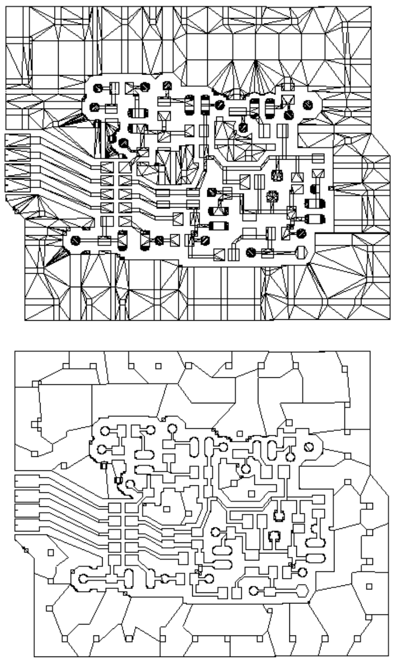

Figure 5. (a) Rectangular-triangular and (b) Polygonal mesh for the RF board interconnection layout

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TABLE 1. Computer resources (PC-NT Pentium III, 700 MHz)

mesh	rectangular/triangular	polygonal
matrix size	3428	733
process size	152.48 MB	59.35 MB
user time	3h 14m 51s	14m 24s

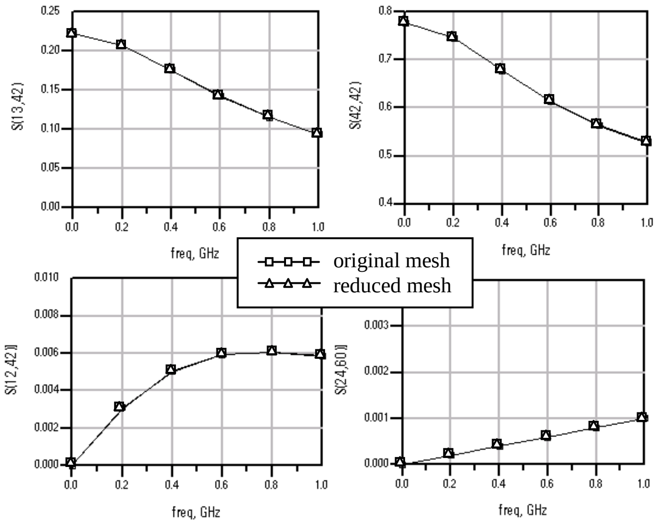

Figure 6. S-parameter results for the RF board interconnection structure ($S_{12,42}$, $S_{13,42}$, $S_{42,42}$ and $S_{24,60}$).

IV CONCLUSION

We introduced new polygonal vector functions that are the generalization of the rectangular and triangular rooftop functions. They allow for a more flexible meshing of complex geometrical structures. The generalized polygonal functions satisfy the current continuity relation and are therefore very suited to model current flow in planar structures. When applied in a planar electromagnetic simulator, they result in a significant performance enhancement for complex geometrical structures as illustrated in the numerical example.

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